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HEAT TRANSFER STABILITY IN MATERIALS PROCESSING. FINITE DIFFERENCE/GALERKIN METHOD

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ABSTRACT

The paper is a study of the active control of heat transfer stability in materials processing using the finite difference/Galerkin method. Using the benefits of this method (simplicity, clearness e.a.), the paper presents the reduction of convection process in a fluid layer heated from bellow by a constant heat flux and cooled from above by convection using a linear proportional control method associated to a shadowgraphic measurement system.

1. Introduction

The active control of heat transfer stability in fluid layers is an intensively studied problem. The researchers established that the onset of Rayleigh-Bénard convection (where the gravitational field is responsible for the convection process) could be delayed using active reactive feedback control systems [1÷3]. It was proved analytically and experimentally and the process is still treated in research activities all over the world.

Instead, the active control of Bénard-Marangoni, where, in the absence of gravity, the surface tension is the diving mechanism [4÷6], was analyzed only analytically, after the process of Bénard-Marangoni convection was proved by space experiments made on Apollo13 and Apollo 17 flights.

The reduction or elimination of Bénard-Marangoni convection process in a fluid layer heated from bellow with a constant heat flux and cooled from above by convection was studied previously [8]. The numerical method was the Fourier decomposition of velocity and temperature fields and it shows the possibility of eliminating the Bénard-Marangoni convection using an active reactive feedback control system. A disadvantage of the method is the big number of (physical and computational) variables the researcher has to keep in the computer memory.

Another computational method used for studying the Bénard-Marangoni convection of a fluid layer is the finite difference/Galerkin method [9], a method that is combining the finite difference decomposition on the fluid layer height with the Fourier decomposition on the fluid wavelength direction. The method proved itself of giving many calculation advantages: simplicity, less computational operations, e.a [3, 10, 11].

This paper is analyzing the active control of Bénard-Marangoni convection of a fluid layer heated from bellow by a constant heat flux and cooled from above by convection using the finite difference/Galerkin method. The method is associated to a shadowgraphic measurement.

2. Mathematical formulation

The mass, momentum and energy conservation equations for the fluid layer are [12, 13]:

$$\nabla \cdot v' = 0 ; \tag{1}$$

$$\frac{\partial v'}{\partial t'} + v' \cdot \nabla v' = -\frac{1}{\rho_0} \nabla p' - \frac{\rho}{\rho_0} g\bar{z} + \frac{\mu}{\rho_0} \nabla^2 v' ; \tag{2}$$

$$\rho c_p \left(\frac{\partial T'}{\partial t'} + v' \cdot \nabla T' \right) = k \nabla^2 T' , \tag{3}$$

where v' is the velocity field in the horizontal direction (y), w' is the velocity field in the vertical direction (z), T' is the temperature field, t' is time, k is the fluid thermal conductivity, μ is the dynamic viscosity, c_p is the specific heat, ρ is the fluid density, ρ_0 is the fluid density at the reference temperature T_0, p is the pressure, g is the gravitational acceleration.

The following non-dimensional variables are defined:

$$v=\frac{L}{\alpha}v',\ w=\frac{L}{\alpha}w',\ T=\frac{T'}{\frac{qL}{k}},\ p=\frac{L^2}{\alpha\nu\rho_0}p',\quad y=\frac{y'}{L},\ z=\frac{z'}{L},\ t=\frac{\alpha}{L^2}t' \tag{4}$$

for velocity, temperature, pressure, length and time. Here, L is the fluid layer thickness α is the fluid layer thermal diffusivity, ν is the kinematics viscosity, q is the heat flux applied at the lower boundary.

The Boussinesq approximation imposes:

$$\rho=\rho_0(1-\beta(T-T_0)), \tag{5}$$

with β—the volumetric expansion coefficient.

The equations (4), (5) and the system of equations (1)÷(3), lead to the non-dimensional conservation equations, (6)÷(8):

$$\nabla\cdot v=0; \tag{6}$$

$$\left(\frac{\partial v}{\partial t}+v\cdot\nabla v\right)=-\nabla p^{*}-RaT\bar{z}+\nabla^2 v; \tag{7}$$

$$\frac{\partial T}{\partial t}+v\cdot\nabla T=k\nabla^2 T, \tag{8}$$

where $p^{*}=p-p_{hydrostatic}$, Rayleigh number $Ra=\frac{\beta gqL^4}{k\alpha\nu}$, Prandtl number $Pr=\frac{\nu}{k}$.

Equation (7) can be written, as a function of vorticity $\zeta=\frac{\partial w}{\partial y}-\frac{\partial v}{\partial z}$, as follows:

$$\frac{1}{Pr}\left(\frac{\partial\zeta}{\partial t}+v\frac{\partial\zeta}{\partial y}+w\frac{\partial\zeta}{\partial z}\right)=Ra\frac{\partial T}{\partial y}+\nabla^2\zeta. \tag{9}$$

In order to establish the equilibrium temperature field, I am considering Fourier cosine series decomposition for the temperature field (T), the vertical (w) and horizontal (v) velocity fields [3,10]:

$$T(y,z)=T_0(z)+\sqrt{2}\sum_{m=1}^{N}T_m(z)\cos(a_k y); \tag{10}$$

$$w(y,z)=\sqrt{2}\sum_{m=1}^{N}w_m(z)\cos(a_k y); \tag{11}$$

$$v(y,z)=\sqrt{2}\sum_{m=1}^{N}v_m(z)\cos(a_k y), \tag{12}$$

where $a_k=\frac{2\pi k}{\lambda}, k=1\ldots M$, $D=\partial/\partial z$; λ is the ratio fluid length/height, N is the number of elements considered for the Fourier series.

The orthogonal basis of the Galerkin procedure is: 1, $\sqrt{2}\sin(\alpha_m y)$, $\sqrt{2}\cos(\alpha_m y)$.

Next, the equations (10÷12) are substituted in the non-dimensional form of conservation equations (6), (8) and (9). Averaging equation (6) over y direction, equation (13) is obtained:

$$v_m=-\frac{Dw_m}{a_m}. \tag{13}$$

Equation (14) is obtained averaging equation (8) over y:

$$D^2T_0=\sum_{m=1}^{K}(Dw_mT_m+w_mDT_m). \tag{14}$$

Multiplying equation (8) with $\sqrt{2}\cos(\alpha_m y)$ and averaging over y the following, the modal temperature equations are obtained:

$$D^2T_m-(\xi w_{2m})DT_m-\left(\alpha_m^2-\frac{\xi}{2}Dw_{2m}\right)T_m = DT_0+\xi(2T_{2m}Dw_m+DT_{2m}w_m)+\xi f_1 \tag{15}$$

with 2m<N and

$$f_1=\sum_{k=l\neq m}^{K}\sum_{l=l\neq m}^{K}\left(\frac{\alpha_k TkDw_l}{\alpha_l}I_{klm}^{ssc}\right)+\sum_{k=l\neq m}^{K}\sum_{l=l\neq m}^{K}\left(DT_k w_l I_{klm}^{ccc}\right) \tag{16}$$

Similarly, multiplying equation (9) with $\sqrt{2}\sin(\alpha_m y)$ and averaging over y, the modal velocity equation is obtained:

$$\begin{aligned}&D^4w_m-(\phi w_{2m})D^3w_{2m}+\left(\frac{\phi}{2}Dw_{2m}-2\alpha_m^2\right)\\&D^2w_m+\left(\alpha_{2m}^2\phi w_{2m}-\phi D^2w_{2m}-\alpha_m^2\phi w_{2m}\right)\\&Dw_m+\left(\alpha_m^4+\alpha_m\alpha_{2m}\phi Dw_{2m}\right)w_m+\\&\left(-\frac{\phi}{2}D^3w_{2m}-\frac{\alpha_m^2}{2}\phi Dw_{2m}\right)w_m=\\&\alpha_m^2RaT_m+\phi f_2\end{aligned} \tag{17}$$

with 2m<N and

$$f_2=\sum_{k=l\neq m}^{K}\sum_{l=l\neq m}^{K}\left(\frac{\alpha_k^2w_kDw_l}{\alpha_l}-\frac{D^2w_kw_l}{\alpha_l}\right)I_{klm}^{css}+\sum_{k=l\neq m}^{K}\sum_{l=l\neq m}^{K}+\left(\frac{D^3w_kw_l}{\alpha_l}-\alpha_kDw_kw_l\right)I_{klm}^{scs} \tag{18}$$

where $\phi=\frac{1}{\sqrt{2}Pr}$.

3. Numerical method

The equations (14), (15) and (17) were discretized using centered finite differences as indicated following the example set before [8,7,10,11]. Consequently, we have to solve 2N+1 systems of equations. Each system of equation has Nz unknowns, the number of points that were considered for the z decomposition.

In order to solve the systems of equations, the following boundary conditions were considered:

— the no slip (lower boundary) and no penetration (upper and lower boundary) conditions:

$$w_m\big|_{z=0} = w_m\big|_{z=1} = Dw_m\big|_{z=0}\ ; \qquad (19)$$

— the upper boundary heat transfer conditions:

$$DT_m + BiT_m = 0\Big|_{z=1}\ ; \qquad (20)$$

$$T_0 = 1/Bi\Big|_{z=1}, \qquad (21)$$

where Bi is the Biot number.

— the condition for surface tension driven convection at the upper boundary:

$$D^2 w_m = Ma\cdot a^2\cdot(T_m)\Big|_{z=1}, \qquad (22)$$

where Ma is the Marangoni number and a is the wave number, $a = 2\pi/\lambda$.

— at the lower boundary, the applied heat flux is proportional to a local shadowgraphic signal; that signal is giving information on the deviation (in each point) of the temperature field from the equilibrium value (in other words the temperature when only diffusion is responsible for the heat transport); γ is the "proportional gain":

$$DT_0 = -1\Big|_{z=0}\ ; \qquad (23)$$

$$DT_m = \gamma a^2 \int_0^1 T_m\, dz\ . \qquad (24)$$

Taking into consideration the working method and the particularities I mentioned before, the system of equation (14) becomes:

$$(A_0)\cdot(B_0) = (C_0), \qquad (25)$$

$$(A_0)=\begin{pmatrix} \frac{-2}{\Delta z^2} & \frac{1}{\Delta z^2} & & & \\ \frac{1}{\Delta z^2} & \frac{-2}{\Delta z^2} & \frac{1}{\Delta z^2}\cdots & & 0 \\ & \frac{1}{\Delta z^2} & \frac{-2}{\Delta z^2} & \cdots & 0 \\ & \cdots & & & \cdots \\ & \cdots & & \frac{-2}{\Delta z^2} & \frac{1}{\Delta z^2} \\ & \cdots & & \frac{1}{\Delta z^2} & \frac{-2}{\Delta z^2} \end{pmatrix},\ (B_0)=\begin{pmatrix} T_0^2 \\ T_0^3 \\ \cdots \\ T_0^{Nz-4} \\ T_0^{Nz-3} \\ T_0^{Nz-2} \end{pmatrix},\ (C_0)=\begin{pmatrix} B_0^2-\frac{1}{\Delta z} \\ B_0^3 \\ \cdots \\ B_0^{Nz-4} \\ B_0^{Nz-3} \\ B_0^{Nz-2}-\left(\Delta z+\frac{1}{Bi}\right)\frac{1}{\Delta z^2} \end{pmatrix},\ \begin{cases} T_0^1 = T_0^2 + \Delta z \\ T_0^{Nz} = 1/Bi \\ T_0^{Nz-1} = 1/Bi + \Delta z \end{cases}, \qquad (26)$$

$\Delta z = 1.0/(Nz-1)$. The system of equations (15) becomes: $(A_m)\cdot(B_m) = (C_m)$ (27)

$$(A_m)=\begin{pmatrix} \left[\left(\frac{2}{\Delta z}+ter\right)/A-2-\alpha_m^2\right]\frac{1}{\Delta z^2} & \left[\left(\frac{-1}{2\Delta z}+ter\right)/A+1\right]\frac{1}{\Delta z^2} & \frac{-ter}{\Delta z^2 A}\cdots & \frac{ter}{\Delta z^2 A} & \frac{-ter}{\Delta z^2 A}(1{,}5-2Bi\Delta z) \\ \frac{1}{\Delta z^2} & \frac{-2}{\Delta z^2} & \frac{1}{\Delta z^2}\cdots & 0 & 0 \\ 0 & \frac{1}{\Delta z^2} & \frac{-2}{\Delta z^2} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \\ \cdots & \cdots & \cdots\ \left(\frac{1}{\Delta z^2}-\alpha_m^2-\frac{\xi}{2}Dl\right) & \frac{1}{\Delta z^2} & \frac{1+2Bi\Delta z}{\Delta z^2} \\ \cdots & \cdots & \cdots\ 0 & \left(\frac{-2}{\Delta z^2}-\alpha_m^2-\frac{\xi}{2}Dl\right) & \frac{2+2Bi\Delta z}{\Delta z^2} \\ \cdots & \cdots & \cdots\ \frac{-4}{\Delta z^2} & \frac{-5}{\Delta z^2} & \left(\frac{6+8Bi\Delta z}{\Delta z^2}-\alpha_m^2-\frac{\xi}{2}Dl\right) \end{pmatrix} \qquad (28)$$

$$(B_m)=\begin{pmatrix} T_m^2 \\ T_m^3 \\ \cdots \\ T_m^{Nz-3} \\ T_m^{Nz-1} \\ T_m^{Nz} \end{pmatrix},\quad (C_m)=\begin{pmatrix} B_m^2 \\ B_m^3 \\ \cdots \\ B_m^{Nz-3} \\ B_m^{Nz-1} \\ B_m^{Nz} \end{pmatrix}, \qquad (29)$$

with: $ter=\gamma a^2\Delta z$ and $A=3/2/\Delta z$; Dl is the first derivative of the vertical velocity, w_{2m}^1.

$$B_m^i = \sum_{k=1}^{m}\left(T_0^i Dw_k^i + w_k^i DT_0^i\right) \text{ and}$$

$$T_m^1 = -T_0^2\left(\frac{-2}{\Delta z}+ter\right)\frac{1}{A} + T_0^3\left(\frac{1}{2\Delta z}+ter\right)\frac{1}{A} - T_0^{Nz}\frac{ter}{2A} - \sum_{i=4}^{Nz-1} T_0^i \frac{ter}{A}$$

The system of equations (17) becomes:

$$(D_m)\cdot(E_m) = (F_m) \qquad (30)$$

$$
(E_m)=\begin{pmatrix} w_m^3 \\ w_m^4 \\ \dots \\ w_m^{Nz-2} \end{pmatrix},\quad
(F_m)=\begin{pmatrix} A_m^3 \\ A_m^4 \\ \dots \\ A_m^{n-3}+\dfrac{a^2MaT_m^{Nz}}{5\Delta z^2} \\ A_m^{n-2}+\dfrac{4a^2MaT_m^{Nz}}{5\Delta z^2}+\dfrac{2\alpha_m^2-2a^2MaT_m^{Nz}}{5} \end{pmatrix},\quad
\begin{cases} w_m^1=0 \\ w_m^2=w_m^3/4 \\ w_m^{Nz=0} \\ w_m^{Nz-1}=-0{,}4Maa^2T_m^{Nz}\Delta z^2+ \\ \qquad 0{,}8w_m^{Nz-2}\dots \end{cases} \tag{31}
$$

$$
(D_m)=\begin{pmatrix}
\left(\frac{5}{\Delta z^4}+\alpha_m^4+\frac{\phi w_{2m}^3}{4\Delta z^3}+BB-\frac{\alpha_m^2}{8\Delta z^2}+DD\right) & \left(\frac{-4}{\Delta z^4}+BB-\frac{\phi w_{2m}^3}{\Delta z^3}+CC\right) & \left(\frac{1}{\Delta z^4}+\frac{\phi w_{2m}^3}{2\Delta z^3}\right) & 0\dots & & \\
\left(\frac{-15}{\Delta z^4}+BB+\frac{7\phi w_{2m}^4}{8\Delta z^3}\right) & \left(\frac{6}{\Delta z^4}+\alpha_m^4+BB+DD\right) & \left(\frac{-4}{\Delta z^4}+BB-\frac{\phi w_{2m}^3}{\Delta z^3}+CC\right) & \left(\frac{1}{\Delta z^4}-\frac{\phi w_{2m}^4}{2\Delta z^3}\right) & 0\dots & \\
\left(\frac{1}{\Delta z^4}+\frac{\phi w_{2m}^5}{2\Delta z^3}\right) & \left(\frac{-4}{\Delta z^4}+BB-CC+\frac{\phi w_{2m}^5}{2\Delta z^3}\right) & \left(\frac{6}{\Delta z^4}+\alpha_m^4+BB+DD\right) & \left(\frac{-4}{\Delta z^4}+BB+CC+\frac{\phi w_{2m}^5}{2\Delta z^3}\right) & \left(\frac{1}{\Delta z^4}+\frac{\phi w_{2m}^5}{2\Delta z^3}\right) & 0\dots \\
\dots & \dots & \dots & \dots & & \\
\dots & 0 & \left(\frac{1}{\Delta z^4}-\frac{\phi w_{2m}^{Nz-3}}{2\Delta z^3}\right) & \left(\frac{-16/5}{\Delta z^4}-\frac{2\alpha_m^2}{\Delta z^2}\right) & \left(\frac{24/5}{\Delta z^4}+\alpha_m^4+BB+DD\right) & \left(\frac{-16/5}{\Delta z^4}-\frac{2\alpha_m^2}{\Delta z^2}\right) \\
\dots & & 0 & \left(\frac{1}{\Delta z^4}-\frac{\phi w_{2m}^{Nz-2}}{2\Delta z^3}\right) & \left(\frac{-16/5}{\Delta z^4}+\frac{(-8/5)\alpha_m^2}{\Delta z^2}\right) & \left(\frac{14/5}{\Delta z^4}+\alpha_m^4+BB-\frac{(8/5)\alpha_m^2}{\Delta z^2}+DD\right)
\end{pmatrix}, \tag{32}
$$

where $BB=\dfrac{(\phi D1-2\alpha_m^2)}{\Delta z^2}$;

$$CC=\frac{1}{2\Delta z}\left(\alpha_m^4+\phi w_{2m}^3-\phi D2-\phi w_{2m}^3\alpha_m^2\right);$$

$$DD=\frac{1}{2\Delta z}\left(\alpha_m^4+\alpha_m\alpha_{2m}\phi D1-\frac{\phi}{2}D3-\frac{\alpha_m^2}{2}\phi D1\right);$$

D1, D2 and D3 are the first, second and the third derivative of w_{2m}^i, where i is the corresponding point on the z direction. The numerical procedure, the initial guess, the convergence criteria and the relaxation procedure was described elsewhere[].

4. Numerical results.

Throughout the paper Nz=100, N=4, Biot number Bi=1.0, Marangoni number Ma=200.0, Prandtl number Pr=6.7, the wavelength λ=2,464.

I called the working method described above as the "implicit" method. The study of the numerical results of the "implicit" method showed a very sensitive dependence on the number of points used for the discretization in the z direction, Nz. Its level determines the vorticity amplitude and, consequently, the stability level of the fluid layer. The Nz=100 approach (110 points is the maximum value allowed by this method) is allowing us, for N=4 is not enough for the precision required by the treatment of the problem using finite difference/Galerkin procedure.

Fig.1 and Fig.2 are presenting the temperature and vortivity fields for the "implicit" method case, Nz=100, N=4, γ=0.

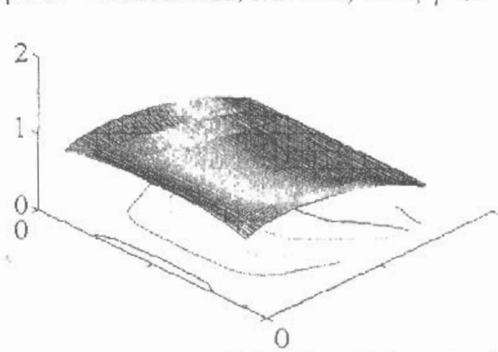

Fig. I Temperature field, "implicit method". Nz=100, N=4, γ=0.

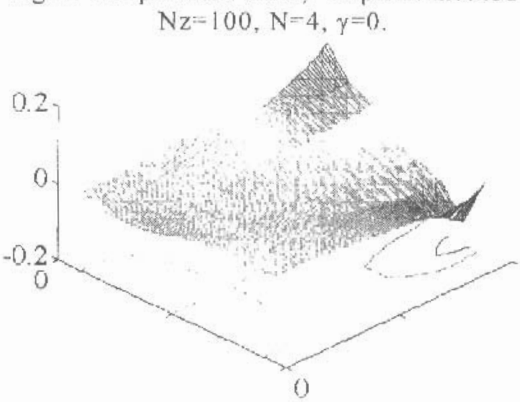

Fig. 2 Vorticity field, "implicit method". Nz=100, N=4, γ=0.

The results obtained for $\gamma=0$ case to not verify the results obtained using the classical method (the Fourier decomposition method [11]). Therefore, I defined a new method, the "explicit" method, where the boundary condition (24) can be written as follows:

$$DT_m = \gamma a^2 \int_0^1 T'_m dz \Big|_{z=0} \qquad (33)$$

where T'_m are the temperature values of the previous iteration step. The equations (28÷29) become:

$$(A_m)=\begin{pmatrix} \left[\frac{2co}{A\Delta z}-\frac{2}{\Delta z^2}-\alpha_m^2\Delta z^2\right] & \left[\frac{-co}{2\Delta zA}+\frac{1}{\Delta z^2}\right] & 0 & \dots & & \\ \frac{1}{\Delta z^2} & \frac{-2}{\Delta z^2} & \frac{1}{\Delta z^2} & 0 & \dots & \\ 0 & \frac{1}{\Delta z^2} & \frac{-2}{\Delta z^2} & 0 & \dots & \\ \dots & \dots & \dots & \dots & & \\ \dots & 0 & \left(\frac{1}{\Delta z^2}-\alpha_m^2-\frac{\xi}{2}Dl\right) & \frac{1}{\Delta z^2} & \frac{1+2Bi\Delta z}{\Delta z^2} & \\ & \dots & 0 & \left(\frac{-2}{\Delta z^2}-\alpha_m^2-\frac{\xi}{2}Dl\right) & \frac{2+2Bi\Delta z}{\Delta z^2} & \\ & \dots 0 & \frac{-4}{\Delta z^2} & \frac{-5}{\Delta z^2} & \left(\frac{6+8Bi\Delta z}{\Delta z^2}-\alpha_m^2-\frac{\xi}{2}Dl\right) & \end{pmatrix} \qquad (34)$$

$$(B_m)=\begin{pmatrix} T_m^2 \\ T_m^3 \\ \dots \\ T_m^{Nz-3} \\ T_m^{Nz-1} \\ T_m^{Nz} \end{pmatrix},\ (C_m)=\begin{pmatrix} B_m^2-\frac{CC}{AA\Delta z^2} \\ B_m^3 \\ \dots \\ B_m^{Nz-3} \\ B_m^{Nz-1} \\ B_m^{Nz} \end{pmatrix}, \qquad (35)$$

$$B_m^i=\sum_{k=1}^{mp}(T_0^i Dw_k^i + w_k^i DT_0^i);$$

$$co=\frac{1}{\Delta z^2}+\xi w_{2m}^i \frac{1}{2\Delta z};\ CC=\gamma a^2 \Delta z \sum_{i=1}^{Nz} T_m^i;$$

$$T_m^1=-T_0^2\left(\frac{-2}{\Delta z}\right)\frac{1}{A}+T_0^3\left(\frac{1}{2\Delta z}\right)\frac{1}{A}-\frac{CC}{A}.$$

This new method is allowing Nz values as high as 280points for N=4. The results difference can be seen by comparing Fig.2 and Fig.3. Fig. 3 is presenting the vorticity field for the same parameters: Ma=200, Bi=1, λ=2,464, but for the "explicit method" solution.

This new method are approaching, for γ=0, the results obtained for the same problem using only Fourier decomposition technique. Fig.4 and Fig.5 are presenting the vorticity and temperature fields for the "explicit" method and a proportional gain γ=10.

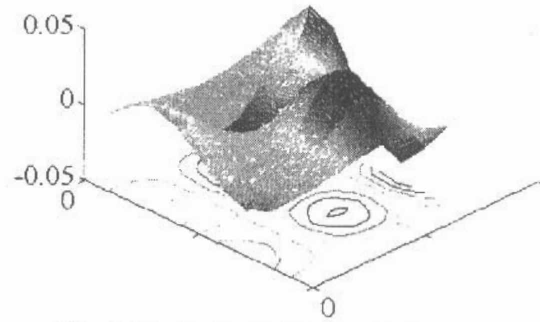

Fig. 3 Vorticity field, "explicit method". Nz=280,N=4, γ=0.

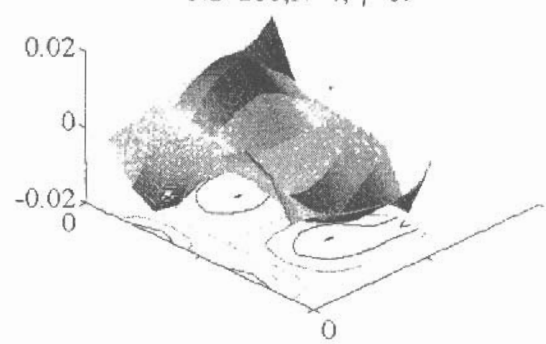

Fig. 4 Vorticity field, "explicit method". Nz=280,N=4, γ=10.

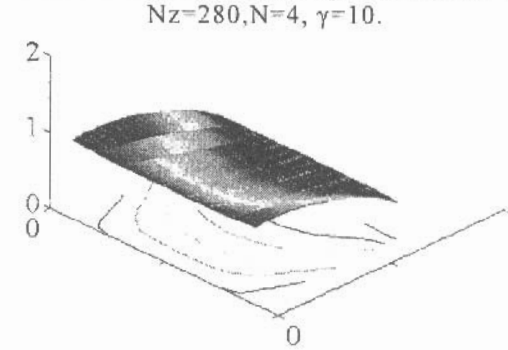

Fig. 5 Temperature field, "explicit method". Nz=280, N=4, γ=10.

The influence of the proportionality factor, γ, on the level of vorticity field, and consequently, the stability of the fluid layer, is presented by Fig.6. It shows the decrease of the vorticity as the proportionality parameter γ increases.

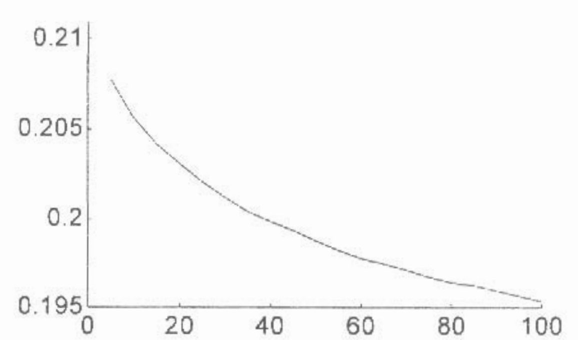

Fig. 6 Vorticity (ξ) — proportional gain (γ) dependence; "explicit method", λ=2,464; Bi=1.

The influence of the wave number on the vorticity level is presented by Fig.7 for two different values of Biot number. Higher values of the wave number imply lower values of the vorticity, keeping all the other parameters constant.

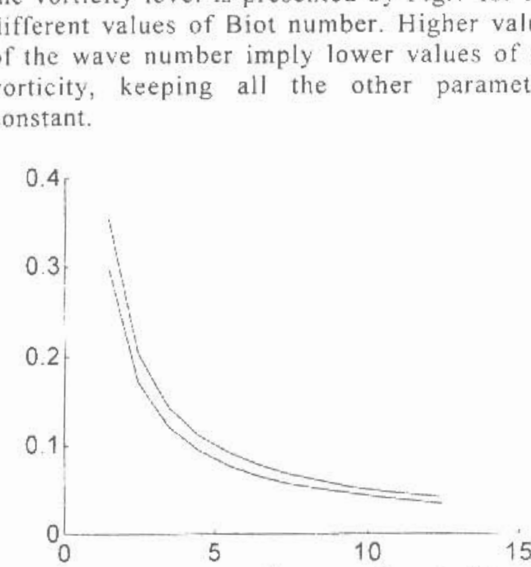

Fig. 7 Vorticity (ξ)— wavelength (λ) dependence, "explicit method"; γ =20; Bi=1, Bi=10.

5. Conclusions

The results presented above are driving us toward the following conclusions:

1. The finite difference/Galerkin method can be successfully used for the theoretical study of heat transfer stability in active control of a fluid layer convection process.
2. The anticipated advantages of this method [3,10,11] proved their utility for the analyzed case(we do not need to store the physical components of the temperature and velocity fields during calculation process and, consequently, we can have more iteration points and a higher precision for the results; the simplicity of the software needed to solve this problem, the decrease both of the calculation operations needed to be done and of the computational time).
3. The results obtained, the vorticity level, depend strongly on the number of points, Nz. This dependence is stronger here than in the previous methods used for solving this problem. This is a disadvantage counting heavily in the economy of the solution.
4. This problem could be solved using the "explicit" method suggested in the paper. Even if this method is presenting itself as being a potential alternative in the study of active control of convection of a fluid layer heated from bellow by a heat flux and cooled from above by convection, it was not able to establish the onset of convection point.
5. The variation of the vorticity field levels as a function of wavelength and proportional gain shows a good trend in the "implicit" method, the vorticity level decreasing as the proportional gain and the wavenumber increase.

These results are emphasizing the potential this method has for further study of active control of Bénard-Marangoni convection.

Bibliography

1. **Tang J. and Bau H. H.**, 1994, *"Stabilization of the No-Motion State in the Rayleigh-Bénard Problem"*, Proc R Soc. Lond. A **447**, 587.
2. **Tang J. and Bau H.H**., 1995, *"Stabilization of the No-Motion State of a Horizontal Fluid Layer Heated from Bellow with Joule Heating"*, J. of Heat Trans. **117**, 329
3. **Howle, L.E.**, *Efficient Implementation of a Finite-Difference/Galerkin Method for Simulation of Large Aspect Ratio Convection*, Numerical Heat Transfer, Part B. **26**, 105 (1994).
4. **Nield, D. A.**, *"Surface tension and buoyancy effect in cellular convection"*, J. Fluid Mech. **19**. 341 (1964).
5. **Scriven, L.E., and Sterling, C.V**., *"On cellular convection driven by surface-tension gradients: effect of mean surface tension and surface viscosity"*, J. Fluid Mech. **19**, 321 (1964)
6. **Smith, K.A.**, *"On convective instability induced by surface tension gradients"*, J. Fluid Mech. **24**, 401 (1966).
7. **M. Neagu**, ***Numerical Modeling of the Temperature Filed in a Melted Material Layer***, National Conference OPROTEH-2001, 291, 2001.
8. **M. Neagu**, ***Active control of Heat Transfer Stability in Materials Processing***, International Conference SIMSIS-11, Galați, Octombrie 2001, 315, 2001.
9. **G. Zilli**, *Metodi Variationali per Equazione Differenziali*, Imprimitur Editrice, Padova, april 1998.
10. **McDonough, J.M., Catton, I**, *A Mixed Finite Difference-Galerkin Procedure for Two-Dimensional Convection in a Square Box*. Int. J. Heat Mass Transfer, **25**, no.8, 1137 (1982).
11. **M. Neagu, G. Frumusanu**, *Finite Difference/Galerkin Solution for a Melted Material Layer Temperature Field*, Analele Universității "Dunărea de Jos" din Galați XIX(XIV), 61, 2001.
12. **Bejan A.**, *Heat Transfer*, Wiley, New York, 1993.
13. **Chandrasekhar, S.**, *"Hydrodynamic and Hydromagnetic Stability"*, Dover, New York, 1981.
14. **L. Czechowski, J.M. Floryan**, *Marangoni Instability in a Finite Container-Transition Between Short and Long Wavelengths Modes*, Transactions od the A.S.M.E. (123), 96, 2001.

STABILITATEA CAMPULUI TERMIC LA PRELUCRAREA MATERIALELOR. METODA "DIFERENTE FINITE/GALERKIN"

Rezumat

Lucrarea este un studiu al controlului activ al stabilitatii termice la prelucrarea materialelor folosind metoda diferente finite/Galerkin. Utilizand avantajele acestei metode(simplitate, claritate, etc), lucrarea prezinta reducerea procesului de convectie intr-un strat de fluid incalzit de un flux constant la granita inferioara si racit prin convectie la cea superioara, utilizand o metoda de control proportionala asociata unui sistem "shadowgrafic" (engl.) de masurare.

STABILITÉ DE LA CHALEUR TRASFER DANS LE TRAITEMENT DE MATÉRIAUX. MÉTHODE FINIE DE DIFFERENCE/GALERKIN

Résumé

Le papier est une étude de la commande active de la stabilité de transfert thermique en matériaux traitant en utilisant la méthode des differences finie/Galerkin. En utilisant les avantages de cette méthode (simplicité, clarté e.a.), le papier présente la réduction de processus de convection d'une couche liquide de chauffage du beuglement par un flux constant de la chaleur et refroidie de ci-dessus par la convection en utilisant une méthode de contrôle proportionnelle linéaire associée à un système shadowgraphic de mesure.